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Note on a Vibratory Phenomenon Arising in Transducer Calibration

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PREFACE

The work in this publication was conducted at the request of the Instrumentation Applications Section in support of the project "Space Shuttle Pogo Pressure Measuring System" for NASA George C. Marshall Space Flight Center.

Dr. H. S. Oser
Chief, Mathematical Analysis Section

Note on a Vibratory Phenomenon Arising in Transducer Calibration

Richard Kraft

By making appropriate physical approximations and idealizations a theoretical explanation is found for a vibratory phenomenon observed in calibrating pressure transducers inside thin liquid filled cylinders. The theoretical explanation requires proving the equivalence of two boundary initial problems which define the vibratory phenomenon. A short, general and complete proof of this equivalence is given.

Key words: boundary initial problems; measurement; pressure transducer; transducer calibration; vibration.

1. Introduction

The purpose of this note is to provide a theoretical explanation of an experiment connected with dynamic calibration of transducers. The dynamic calibration technique is concerned with small disturbances in a liquid filled metal tube that is open at the top and is mounted vertically on a vibration table, see figure 1b. Sound waves are generated in the liquid by vertical sinusoidal motion of the vibration table. The waves in the fluid are picked up by transducers attached along the side of the tube. It is readily observed that the pressures sensed by the transducer is a function of the applied acceleration, liquid density and position along the tube. Further experiments in which the tube was capped at the top and completely filled with liquid, see figure 1a, were conducted in order to study the effect on the transducer of additional steady pressure. The data from this experiment shows (with reasonable experimental error) the interesting phenomenon that the pressure in an open tube of length, L/2, at any height above the vibration table is the same as that in a closed tube of length, L, at the same height above the vibration table when the closed and open tubes are vibrated at the same frequency. We emphasize that the induced vibrations are off resonance. In section 2, we give a mathematic formulation of these experiments which predicts this phenomenon. Since the mathematical description is based on a set of physical assumptions, see hypotheses I-III below. This prediction increases our confidence in these assumptions and provides a basis for interpretation and correlation of experimental data.

2. Analysis

We consider, for the sake of definiteness, the small disturbance motion imparted simultaneously to the fluid in both systems in figure 1 by a vibratory motion, g(t), of the vibration table. Initially both systems are assumed to be at complete rest. The resulting small displacement field $\xi^c(x,t)$, that is setup inside the capped tube is precisely described, in the one dimensional non-dissipative case, using Lagrangian terminology [2, p. 479] by the equations

$$\partial_t^2 \xi^c(x,t) - C^2 \partial_x^2 \xi^c = 0 \qquad (2.1i)$$

for $0 \leq x \leq L$ and $t \geq 0$,

$$\xi^c(x,0) = \partial_t \xi^c(x,0) = 0 \qquad (2.1ii)$$

for $0 \le x \le L$,

$$\xi^c(0,t) = g(t) \tag{2.1iii}$$

for $t \ge 0$,

$$\xi^c(L,t) = g(t) \tag{2.1iv}$$

for $t \ge 0$.

Likewise, the precise description of the field, $\xi^o(x,t)$, inside the open tube is given by the equations

$$\partial_t^2 \xi^o(x,t) - C^2 \partial_x^2 \xi^o(x,t) = 0 \tag{2.2i}$$

for $x \le L/2$ and $t \ge 0$,

$$\xi^o(x,0) = \partial_t \xi^o(x,0) = 0 \tag{2.2ii}$$

for $0 \le x \le L/2$,

$$\xi^o(0,t) = g(t) \tag{2.2iii}$$

for $t \ge 0$,

$$\partial_x \xi^o(L/2,t) = 0 \tag{2.2iv}$$

for $t \ge 0$.

In these equations $C^2 \equiv \left.\frac{dp}{d\rho}\right|_{\rho_0}$, where a zero subscript denotes an equilibrium value and $g(t)$ is sufficiently smooth to guarantee the existence of a twice continuously differentiable solution of the above equations.

The justification for the boundary conditions employed in the above equation derives from the following premises.

HYPOTHESIS I: The fluid at both ends of the closed tube and at the bottom of the open tube does not separate from the end of the tube. Hence the liquid at the capped ends of both tubes undergoes the same displacements as the end of the tube. This hypothesis can be rationalized by assuming "no-slip" conditions hold at the closed ends of the tube.

HYPOTHESIS II: The metal tubes are perfectly rigid, i.e. they do not vibrate, moreover they are rigidly attached to the vibrating table. In this situation the tube and vibrating table move together as a rigid body, with the same vibratory motion as the vibrating table. This is, in fact, approximately the case, because the wavelengths of sound in the tube is larger than in the liquid, and since the experiments were run below the fundamental resonance of the liquid itself, the entire length of the tube is only a fraction of a wavelength and therefore all parts of it vibrate approximately in phase.

Hypotheses I and II are the justification for boundary conditions (2.1iii,iv) and (2.2iii).

HYPOTHESIS III: The liquid at the open end of the uncapped tube behaves like a free surface, i.e. supports no stress.

This hypothesis implies

$$\partial_x \xi^0(L/2,t) = 0$$

at the open end of the uncapped tube.

Since (in Lagrangian terminology) a small disturbance pressure field, $p - p_o$, is related to its associated displacement field, $\xi(x,t)$, by the formula $p - p_o = K \frac{\partial \xi}{\partial x}$, the experimental phenomenon described in section 1 is implied by the relationship

$$\xi^c(x,t) = \xi^o(x,t) \tag{2.3}$$

$$\text{for} \quad 0 \le x \le L/2 \quad \text{and for } t \ge 0 ,$$

here $\xi^c(x,t)$, $\xi^o(x,t)$ are the fields determined by (2.1i-iv) and (2.2i-iv). That (2.3) is implied by the above equations can be seen in the following way.

First, a function $\xi^*(x,t)$ is defined in the whole interval $0 \le x \le L$, in terms of the twice continuously differentiable solution of (2.2i-iv) namely $\xi^o(x,t)$ by the reflection rule

$$\xi^*(x,t) \equiv \begin{cases} \xi^o(x,t) & \text{for} \quad 0 \le x \le L/2 \\ \xi^o(L - x,t) & \text{for} \quad L/2 \le x \le L . \end{cases} \tag{2.4}$$

It is readily verified using the properties of $\xi^o(x,t)$ that ξ^* is a twice continuously differentiable function in the interval $0 \le x \le L$. Also, since (2.4) implies

$$\partial_t^2 \, \xi^*(x,t) = \begin{cases} \partial_t^2 \, \xi^o(x,t) & \text{for} \quad 0 \le x \le L/2 \\ \partial_t^2 \, \xi^o(L - x,t) & \text{for} \quad L/2 \le x \le L \end{cases}$$

and

$$\partial_x^2 \, \xi^*(x,t) = \begin{cases} \partial_x^2 \, \xi^o(x,t) & \text{for} \quad 0 \le x \le L/2 \\ \partial_x^2 \, \xi^o(L - x,t) & \text{for} \quad L/2 \le x \le L , \end{cases}$$

equation (2.2i) implies $\xi^*(x,t)$ satisfies (2.1i) in the interval $0 \le x \le L$. By using (2.4) to evaluate the quantities $\xi^*(0,t)$ and $\xi^*(L,t)$ it is seen that $\xi^*(x,t)$ satisfies the boundary conditions (2.1iii) and (2.1iv). Finally, in the same way (2.4) implies $\xi^*(x,0)$ satisfies the initial condition (2.1ii). Since it has now been shown that $\xi^*(x,t)$ is twice continuously differentiable and satisfies (2.1i-iv), it must be the unique solution of these equations. Hence $\xi^*(x,t) = \xi^c(x,t)$ and (2.3) follows from this by employing (2.4) again.

We note that the proof of (2.3) goes through in the same way when general initial conditions are substituted into (2.1ii, 2.2ii) and even if a Rayleigh type dissipative term i.e. $-K^2 \partial_t \xi$ is inserted in the wave equations.

In conclusion we remark that the boundary initial problems that determine $\xi^c(x,t)$ and $\xi^o(x,t)$ can be directly solved "by the variation of parameter method" and (2.3) is seen to be true by this method also. However, this direct method of establishing (2.3) is considerably longer and more complicated than the proof above.

3. Discussion

The key part in the previous derivation of relation (2.3) was played by the boundary condition (2.2iv) as this was the condition that suggested and permitted the reflection type definition of ξ^* given in (2.4). The reflection rule leads one to consider the extent to which the relation $\xi^c(0,t) = \xi^c(L,t)$ holds and as stated in hypothesis II there is good reason to suppose it is a good approximation. With a knowledge of this relation and (2.4) the completion of the derivation of (2.3) is straightforward. The physical meaning of the condition (2.2iv) is that the fluid at the top of the open tube is completely free to move and thus no stress, which is proportional to $\frac{\partial \xi^0}{\partial x}$, can be induced in it. Thus, although $\frac{\partial \xi^0}{\partial x}$ is proportional to the fluid pressure, it is misleading to assert that $\frac{\partial \xi^0}{\partial x} = 0$ is true because the atmosphere is in some way forcing the fluid at the fluid-air interface to remain in equilibrium. Indeed, the atmosphere has no appreciable effect on the fluid and that is why it behaves as a free surface.

Experiments aimed at measuring the speed of sound in fluids have been made in fluid filled cylinders [1 and earlier citations therein] and it has been observed that the elasticity of the tube wall effects these measurements. The above analysis shows that the phenomenon described in this note will be disturbed to the extent that such elastic effects are present.

In closing it is noted that the Lagrangian approach used in section 2 is advantageous because the boundary conditions are most naturally described in terms of displacements, and displacement is the chief dependent variable in the Lagrangian formalism.

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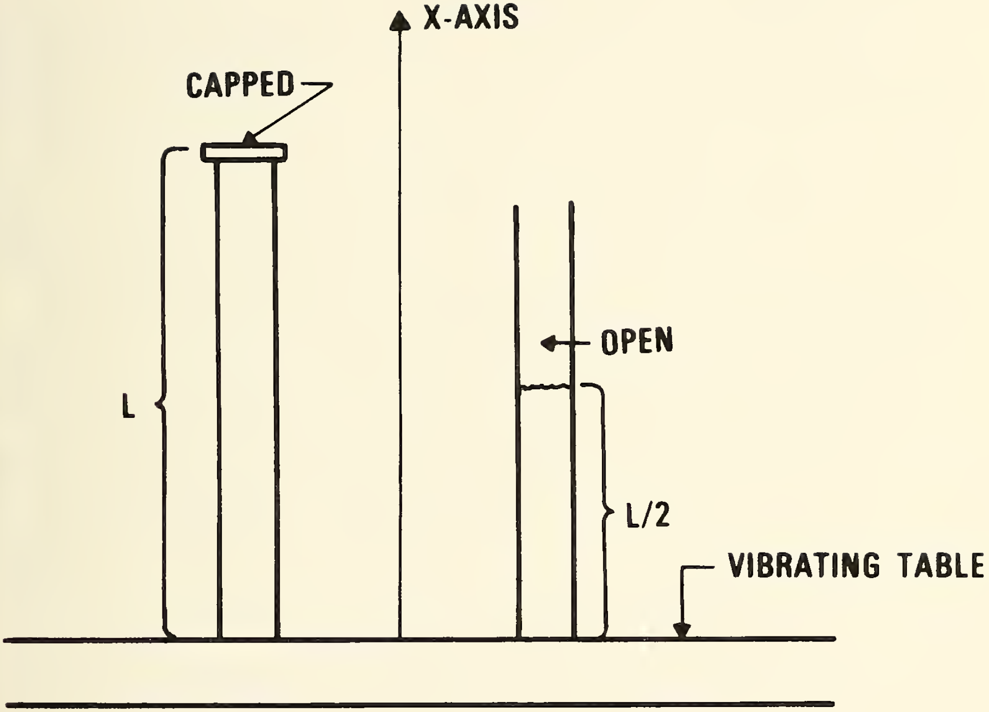

Figure 1a, b: Vibrating cylinders

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